Closed Orbit Errors[Co1,Sa1,Wi2]

A single point kick of strength, ψ, at s = 0 along the circumference of a storage ring gives rise to a closed orbit displacement at position s,

$$\zeta(s) = \frac{\psi}{2}\sqrt{\beta(0)\beta(s)}\,\frac{\cos[\phi(s) - \pi\nu]}{\sin \pi\nu}, \qquad \zeta = x \text{ or } y,$$

where $\beta(0)$ and $\beta(s)$ are the betatron functions at the location of the kick and the observation point respectively, $\phi(s)$ is the phase advance from the kick to the observation point and ν is the betatron tune.

The angular deviation is obtained simply by differentiation,

$$\zeta'(s) = \frac{\psi}{2}\sqrt{\frac{\beta(0)}{\beta(s)}}\,\frac{\sin[\phi(s) - \pi\nu] - \alpha(s)\cos[\phi(s) - \pi\nu]}{\sin \pi\nu},$$

where $\alpha(s) = -\beta'(s)/2$.

Kicks can arise from dipole trim magnets or errors in the main magnets. The table below lists some of the kicks due to magnet errors assuming the betatron phase advance across the displaced element is small.

Element Type	**Source of Kick**	**y**	**Plane**
Quad of Length L & Strength K^2	Displacement by $\Delta_{x,y}$	$K^2L\Delta_{x,y}$	x,y
Dipole of Angle ϕ	Rotation by θ	$\phi\theta$	y
Dipole of Angle ϕ	Field Error, $\frac{\Delta B}{B}$	$\phi\frac{\Delta B}{B}$	x

If the phase advance along the kick is not small, i.e., $\psi(s') \neq \psi\delta(s')$, the closed orbit must be determined from the more general equation for an extended kick,

$$\zeta(s) = \frac{\sqrt{\beta(s)}}{2\sin \pi\nu}\int_C \psi(s')\sqrt{\beta(s')}\cos[\phi(s) - \phi(s') - \pi\nu]\,ds'.$$

The closed orbit blows up for integer tunes indicating the existence of a resonance for ν = integer.

The betatron tune shift due to a gradient error δK^2 is given by

$$\Delta\nu_i = \frac{1}{4\pi}\int \beta_i(s')\,\delta K^2(s')\,ds'.$$

The change in the betatron function is given by

$$\Delta\beta(s) = \frac{\beta(s)}{2\sin 2\pi\nu}\int \beta(s')\,\delta K^2(s')\cos 2\left[\phi(s) - \phi(s') - \pi\nu\right]ds'.$$

The betatron functions blow up for half integer tunes indicating the existence of a resonance for ν = integer/2.

Random Errors: Closed Orbit Amplification Factors[Co1,Gy1]

P_x and P_y are defined to be the ratio between the closed orbit distortion at a particular location which will not be exceeded with 98% probability, to the ‘rms error’ in alignment of the elements.

For quadrupoles, the error is the rms displacement of the element assumed to be the same for all quadrupoles,

$$P_{x,y}^{quad}(s) = \frac{\sqrt{\beta_{x,y}(s)}}{\sin \pi\nu_{x,y}}\left[\sum_{quads} K_j^4 L_j^2 \left\langle \beta_{x,y} \right\rangle_j\right]^{1/2}.$$

For dipoles of bend angle ϕ_j, P_x is the 98% ratio between closed orbit distortions in meters and the relative rms field errors $\Delta B/B$; similarly P_y is the ratio between closed orbit distortions in meters and the rms tilt of the dipoles,

$$P_{x,y}^{dip}(s) = \frac{\sqrt{\beta_{x,y}(s)}}{\sin \pi\nu_{x,y}}\left[\sum_{dips} \phi_j^2 \left\langle \beta_{x,y} \right\rangle_j\right]^{1/2}.$$